1 Presburger Arithmetic: Formal Theory Description

1. $\forall x : \neg(0 = x + 1)$
2. $\forall x \forall y : \neg(x = y) \Rightarrow \neg(x + 1 = y + 1)$
3. $\forall x : x + 0 = x$
4. $\forall x \forall y : (x + y) + 1 = x + (y + 1)$
5. If $P(x)$ is any formula involving the constants $0, 1, +, =$ and a single free variable x, then the following formula is an axiom:
$$(P(0) \wedge \forall x : P(x) \Rightarrow P(x + 1)) \Rightarrow \forall x : P(x)$$

Not that such concepts as divisibility of prime numbers cannot be formalized in Presburger arithmetic. Here is a typical theorem that can be proven from the above axioms:

$$\forall x \forall y : ((\exists z : x + z = y + 11) \Rightarrow (\forall z : \neg(((1 + y) + 1) + z = x)))$$